Scalar model of the glueball

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Abstract

A scalar model of the glueball is offered. The model is based on the nonperturbative calculation of 2 and 4-points Green's functions. Approximately they can be expressed via a scalar field. On the basis of the SU(3) Yang-Mills Lagrangian an effective Lagrangian for the scalar field is derived. The corresponding field equations are solved for the spherically symmetric case. The obtained solution is interpreted as a bubble of the SU(3) quantized gauge field.

1 Introduction

The nonlinearity of quantum chromodynamics leads probably to such objects as a hypothesized flux tube filled with a longitudinal color electric field and stretched between quark and antiquark and a glueball which is a blob of gluonic fields. There is a difference between flux tube and glueball: the first object is created by the quark and antiquark (the sources of the color field) the second one has not any sources - it is a blob of selfinteracting fields. The flux tube has the directed electro-color field but for the glueball it is not clear: whether is it something like monopole or no. These objects are impossible in linear theories, for instance in Maxwell electrodynamics, as its existence is connected with a nonlinear fields interaction among themselves.

At the moment there are different glueball models such as flux tube [1], bags [2], constituent gluons [3], QCD Hamiltonian in Coulomb gauge [4], non-Abelian Born-Infeld theory [5], or the conjectured duality between supergravity and large-N gauge theories [6].

Here we present the glueball model where the gluon field is completely quantum one and it is described by the nonperturbative manner. In this model the gauge field A^B_μ is organized by such a way that $\left\langle A^B_\mu \right\rangle = 0$ but $\left\langle \left(A^B_\mu\right)^2 \right\rangle \neq 0$. In this case the color field in glueball performs nonlinear oscillations and the nonlinearity of the Yang-Mills equations do not allow us to present these oscillations as quanta, i.e. *the glueball is not a cloud of quanta*. The basis for the presented glueball model is: (a) the initial SU(3) Lagrangian for the quantized field $\widehat{A}^B_\mu$ is averaged over some quantum state $|Q\rangle$; (b) the 2 and 4-points Green's functions $\left(\left\langle A^B_\mu A^C_\nu \right\rangle \text{ and } \left\langle A^B_\mu A^C_\nu A^D_\alpha A^E_\beta \right\rangle\right)$ arising in this case are approximated with help of some multiplet of scalar fields; (c) varying with respect to these scalar fields give rise to equations which describe these Green's functions.

2 $A^B_\mu \to \phi^B$ approximation

In any quantum field theory the Green's functions give us the full information about quantized fields. In this section we would like to present equations which will describe 2 and 4-points Green's functions by some approximate manner in the QCD. For this we will average the SU(3) Lagrangian where we use some approximate expressions for 2 and 4-points Green's functions. The SU(3) Lagrangian is

$$\widehat{\mathcal{L}}_{SU(3)} = \frac{1}{4} \widehat{F}^A_{\mu\nu} \widehat{F}^{A\mu\nu} \tag{1}$$

where $\widehat{F}^B_{\mu\nu} = \partial_\mu \widehat{A}^B_\nu - \partial_\nu \widehat{A}^B_\mu + g f^{BCD} \widehat{A}^C_\mu \widehat{A}^D_\nu$ is the field strength operator; $B, C, D = 1, \ldots, 8$ are the SU(3) color indices; g is the coupling constant; f^{BCD} are the structure constants for the SU(3) gauge

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group; $\widehat{A}^B_\mu$ is the gauge potential operator. In order to derive equations describing the quantized field we average the Lagrangian over a quantum state $|Q\rangle$

$$
\begin{aligned}
\left\langle Q \left| \widehat{\mathcal{L}}_{SU(3)}(x) \right| Q \right\rangle = \left\langle \widehat{\mathcal{L}}_{SU(3)} \right\rangle = & \frac{1}{2} \left\langle \left(\partial_\mu \widehat{A}^B_\nu(x) \right) \left(\partial^\mu \widehat{A}^{B\nu}(x) \right) - \left(\partial_\mu \widehat{A}^B_\nu(x) \right) \left(\partial^\nu \widehat{A}^{B\mu}(x) \right) \right\rangle + \\
& \frac{1}{2} g f^{BCD} \left\langle \left(\partial_\mu \widehat{A}^B_\nu(x) - \partial_\nu \widehat{A}^B_\mu(x) \right) \widehat{A}^{C\mu}(x) \widehat{A}^{D\nu}(x) \right\rangle + \\
& \frac{1}{4} g^2 f^{BC_1D_1} f^{BC_2D_2} \left\langle \widehat{A}^{C_1}_\mu(x) \widehat{A}^{D_1}_\nu(x) \widehat{A}^{C_2\mu}(x) \widehat{A}^{D_2\nu}(x) \right\rangle .
\end{aligned} \tag{2}
$$

One can see that schematically we have the following 2, 3 and 4-points Green's functions: $\left\langle (\partial A)^2 \right\rangle$, $\left\langle (\partial A) A^2 \right\rangle$ and $\left\langle (A)^4 \right\rangle$. At first we suppose that the odd Green's functions can be written as the following product

$$
\left\langle \widehat{A}^B_\alpha(x) \widehat{A}^C_\beta(y) \widehat{A}^D_\gamma(z) \right\rangle \approx \left\langle \widehat{A}^B_\alpha(x) \widehat{A}^C_\beta(y) \right\rangle \left\langle \widehat{A}^D_\gamma(z) \right\rangle + \text{(other permutations)} = 0 \tag{3}
$$

as we have supposed that $\langle \widehat{A}^B_\alpha(x) \rangle = 0$. Later we suppose that 2-point Green's function can be presented in so called one-function approximation [9] as

$$
\left\langle \widehat{A}^B_\alpha(x) \widehat{A}^C_\beta(y) \right\rangle = \mathcal{G}^{BC}_{\alpha\beta}(x, y) \approx -\eta_{\alpha\beta} f^{BAD} f^{CAE} \phi^D(x) \phi^E(y) \tag{4}
$$

where $\phi^A(x)$ is the scalar field which describes the 2-point Green's function. The 4-point Green's function can be written in one-function approximation as the product of corresponding two 2-point Green's functions

$$
\begin{aligned}
\left\langle \widehat{A}^B_\alpha(x) \widehat{A}^C_\beta(y) \widehat{A}^D_\gamma(z) \widehat{A}^R_\delta(u) \right\rangle \approx & \left\langle \widehat{A}^B_\alpha(x) \widehat{A}^C_\beta(y) \right\rangle \left\langle \widehat{A}^D_\gamma(z) \widehat{A}^R_\delta(u) \right\rangle + \\
\left\langle \widehat{A}^B_\alpha(x) \widehat{A}^D_\gamma(z) \right\rangle \left\langle \widehat{A}^C_\beta(y) \widehat{A}^R_\delta(u) \right\rangle + & \left\langle \widehat{A}^B_\alpha(x) \widehat{A}^R_\delta(u) \right\rangle \left\langle \widehat{A}^C_\beta(y) \widehat{A}^D_\gamma(z) \right\rangle .
\end{aligned} \tag{5}
$$

Taking into account these expression for the 2,3 and 4-points Green's functions we can derive an effective Lagrangian $\mathcal{L}_{eff} = \left\langle \widehat{\mathcal{L}} \right\rangle$ for the scalar field ϕ^A which describes 2 and 4-points Green's functions (for details, see Appendix A)

$$
\begin{aligned}
\mathcal{L}_{eff} = & -\frac{1}{2} \left(\partial_\mu \phi^A \right)^2 + \frac{\lambda_1}{4} \left[\phi^a \phi^a - \phi^a_0 \phi^a_0 \right]^2 - \frac{\lambda_1}{4} \left(\phi^a_0 \phi^a_0 \right)^2 + \\
& \frac{\lambda_2}{4} \left[\phi^m \phi^m - \phi^m_0 \phi^m_0 \right]^2 - \frac{\lambda_2}{4} \left(\phi^m_0 \phi^m_0 \right)^2 + \left(\phi^a \phi^a \right) \left(\phi^m \phi^m \right)
\end{aligned} \tag{6}
$$

where the indices $a = 1, 2, 3$ are $SU(2)$ indices, $m = 4, 5, 6, 7, 8$ are the coset $SU(3)/SU(2)$ indices, ϕ^A_0 are some constants. The field equations are

$$
\partial_\mu \partial^\mu \phi^a = -\phi^a \left[2\phi^m \phi^m + \lambda_1 \left(\phi^a \phi^a - \phi^a_0 \phi^a_0 \right) \right], \tag{7}
$$

$$
\partial_\mu \partial^\mu \phi^m = -\phi^m \left[2\phi^a \phi^a + \lambda_2 \left(\phi^m \phi^m - \phi^m_0 \phi^m_0 \right) \right]. \tag{8}
$$

One can note that in Ref. [7] was investigated a system of coupled scalar fields and was shown that such system may have soliton solutions by some specific choice of the potential term.

3 Glueball as a bubble of ϕ^A field

Now we would like to consider the spherically symmetric solution with the following ansatz for the scalar field

$$
\phi^a(r) = \frac{\phi(r)}{\sqrt{6}}, \quad a = 1, 2, 3, \tag{9}
$$

$$
\phi^m(r) = \frac{f(r)}{\sqrt{10}}, \quad m = 4, 5, 6, 7, 8. \tag{10}
$$

Let us note that this ansatz means that the components ϕ^a have another behaviour then the components ϕ^m. One can say that such situation is close to a colored flux tube [8] solution filled with the longitudinal

electric field. After substitution (9) (10) into equation (7) (8) we have

$$\phi'' + \frac{2}{r}\phi' = \phi\left[f^2 + \lambda_1\left(\phi^2 - m^2\right)\right], \tag{11}$$

$$f'' + \frac{2}{r}f' = f\left[\phi^2 + \lambda_2\left(f^2 - \mu^2\right)\right] \tag{12}$$

where $2\phi_0^a\phi_0^a = m^2$ and $2\phi_0^m\phi_0^m = \mu^2$; m, μ are some constants which will be calculated by solving equations (11) and (12) and we redefine $\lambda_{1,2}/2 \to \lambda_{1,2}$. Evidently these equations can not be calculated analytically. The preliminary numerical investigations show that this equations set do not have regular solutions by arbitrary choice of m, μ parameters. We will solve equations (7) (8) as a nonlinear eigenvalue problem for eigenstates $\phi(x), f(x)$ and eigenvalues m, μ, i.e. we will calculate m, μ parameters such that the regular functions $\phi(r)$ and $f(r)$ do exist.

At first we note that the solution depends on the following parameters: $\phi(0), f(0)$ and $\lambda_{1,2}$. We can decrease the number of these parameters dividing equations (11) (12) on $\phi^3(0)$. After this we introduce the dimensionless radius $x = r\phi(0)$ and redefine $\phi(x)/\phi(0) \to \phi(x)$, $f(x)/\phi(0) \to f(x)$ and $m/\phi(0) \to m$, $\mu/\phi(0) \to \mu$. Thus we have the following equations set

$$\phi'' + \frac{2}{x}\phi' = \phi\left[f^2 + \lambda_1\left(\phi^2 - m^2\right)\right], \tag{13}$$

$$f'' + \frac{2}{x}f' = f\left[\phi^2 + \lambda_2\left(f^2 - \mu^2\right)\right]. \tag{14}$$

We will search the regular solution with the following boundary conditions

$$\phi(0) = 1, \quad \phi(\infty) = m, \tag{15}$$

$$f(0) = f_0, \quad f(\infty) = 0. \tag{16}$$

One can say that $\phi(x)$ is like to kink and $f(x)$ to soliton. Let us rewrite equation (14) in the following form

$$-\left(f'' + \frac{2}{x}f'\right) + fV_{eff} = \left(\lambda_2\mu^2\right)f \tag{17}$$

where we have introduced an effective potential

$$V_{eff} = \left(\phi^2 + \lambda_2 f^2\right). \tag{18}$$

Immediately we see that with the boundary conditions (16) equation (17) is the Schrödinger equation and it may have a regular solution only if V_{eff} has a hole ($\phi^2 \stackrel{x\to\infty}{\longrightarrow} const, f^2 \stackrel{x\to\infty}{\longrightarrow} 0$) and an energy level $\lambda_2\mu^2$ have to be quantized.

4 Numerical solution

We choose the following numerical method for solving equations (13) (14): we take a null approximation for the function $f(x)$ (which is $f_0(x)$) and solve equation (13) in the followng form

$$\phi_0'' + \frac{2}{x}\phi_0' = \phi_0\left[f_0^2 + \lambda_1\left(\phi_0^2 - m_0^2\right)\right] \tag{19}$$

where m_0 is the null approxiamtion for the parameter m, the boundary conditions are (15) and the function $\phi_0(x)$ is zero approximation for the function $\phi(x)$. Having the regular solution $\phi_0(x)$ we can substitute it into equation (14) and solve the equation

$$f_1'' + \frac{2}{x}f_1' = f_1\left[\phi_0^2 + \lambda_2\left(f_1^2 - \mu_1^2\right)\right] \tag{20}$$

with the boundary conditions (16) (for the numerical calculations presented here we take $f_0 = \sqrt{0.6}$). Thus we have the first approximation $f_1(x)$ which we substitute into equation (13)

$$\phi_1'' + \frac{2}{x}\phi_1' = \phi_1\left[f_1^2 + \lambda_1\left(\phi_1^2 - m_1^2\right)\right]. \tag{21}$$

This equation gives us the first approximation for the function $\phi_1(x)$ and so on. On the i^{th} step we will have

$$\phi_i'' + \frac{2}{x}\phi_i' = \phi_i\left[f_{i-1}^2 + \lambda_1\left(\phi_i^2 - m_i^2\right)\right] \tag{22}$$

and

$$f_i'' + \frac{2}{x} f_i' = f_i \left[\phi_i^2 + \lambda_2 \left(f_i^2 - \mu_i^2 \right) \right]. \tag{23}$$

For every step we have the values m_i^2 and μ_i^2 as an approximation for the true eigenvalues values ${m^*}^2$ and ${\mu^*}^2$.

4.1 The more detailed description of the numerical calculations

At first we will describe the numerical solution of equation (19). For this we choose the null approximation for $f(x)$ as

$$f_0(x) = \frac{\sqrt{0.6}}{\cosh^2 \frac{x}{4}}. \tag{24}$$

The typical solutions for the arbitrary value of m_0 are presented on Fig.1. We see that by $m_0 < m_0^*$ (m_0^* is an unknown parameter which gives us the regular solution) the solution $\phi_0(x)$ is singular and near to the singularity the equation has the form

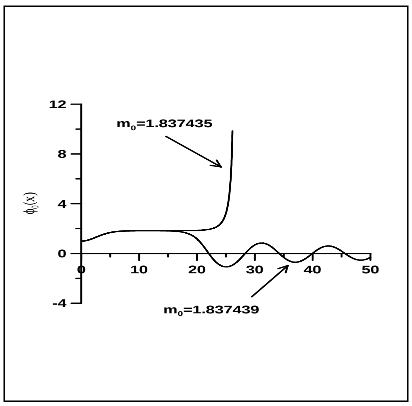

Figure 1: The typical singular solutions for equation (19). $\lambda_1 = 0.1$.

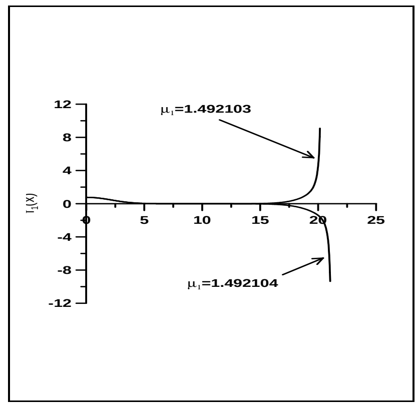

Figure 2: The typical singular solutions for equation (20). $\lambda_2 = 1.0$.

$$\phi_0'' \approx \lambda_1 \phi_0^3 \tag{25}$$

consequently the solution is

$$\phi_0(x) \approx \sqrt{\frac{2}{\lambda_1}} \frac{1}{x_0 - x} \tag{26}$$

where x_0 is some constant depending on m_0. On the other hand by $m_0 > m_0^*$ the solution is presented on Fig.1 and the corresponding asymptotical equation is

$$\phi_0''(x) + \frac{2}{x} \phi_0' \approx - \left(\lambda_1 m^2 \right) \phi_0 \tag{27}$$

which has the following solution

$$\phi_0(x) \approx \phi_\infty \frac{\sin \left(x \sqrt{\lambda_1 m^2} + \alpha \right)}{x} \tag{28}$$

where ϕ_∞ and α are some constants. All of that allows us to assert that there is a value m_0^* for which does exist an exceptional solution which with some accuracy is presented on Fig.4. For this value m_0^* the equation (19) has the following asymptotical behaviour

$$\phi_0''(x) + \frac{2}{x} \phi_0' \approx 2\lambda_1 \left(m_0^* \right)^2 \left(\phi_0 - m_0^* \right) \tag{29}$$

and the corresponding asymptotical solution is

$$\phi_0(x) \approx m_0^* + \beta_0 \frac{e^{-x\sqrt{2\lambda_1 \left(m_0^* \right)^2}}}{x} \tag{30}$$

where ϕ_∞ and β are some constants.

The next step is finding the first approximation for the $f_1(x)$ function. The equation is

$$f_1'' + \frac{2}{x} f' = f_1 \left[\phi_0^2 + \lambda_2 \left(f_1^2 - \mu_1^2 \right) \right]. \tag{31}$$

From the previous calculations one can assume that there is the exceptional regular solution $\phi_0(x)$ with the asymptotical behaviour (30). Then the numerical investigation shows that for the arbitrary μ there are two different singular solutions which are presented on Fig.2. Analogously to equation (25) the

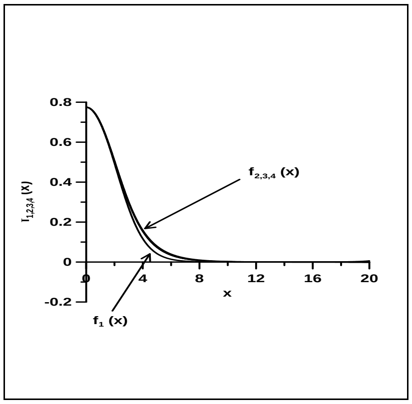

Figure 3: The iterative functions $f_{1,2,3,4}(x)$.

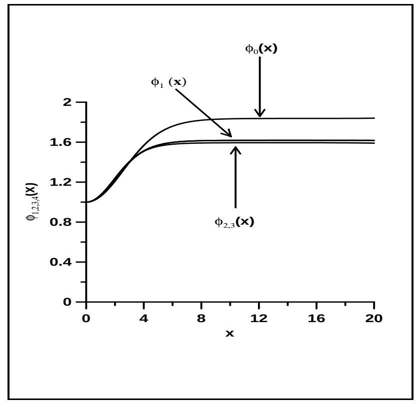

Figure 4: The iterative functions $\phi_{1,2,3,4}(x)$.

singular behaviour of the function $f_1(x)$ is

$$f_1(x) \approx \sqrt{\frac{2}{\lambda_2}} \frac{1}{x - x_0} \quad \text{by} \quad \mu_1 < \mu_1^*, \tag{32}$$

$$f_1(x) \approx -\sqrt{\frac{2}{\lambda_2}} \frac{1}{x - x_0} \quad \text{by} \quad \mu_1 > \mu_1^*. \tag{33}$$

Evidently that we can suppose that there is a regular exceptional solution $f_1^*(x)$ by $\mu_1 = \mu_1^*$ with the following asymptotical behaviour

$$f_1^*(x) \approx f_{\infty,1} \frac{e^{-x\sqrt{\left(m_0^*\right)^2 - \lambda_2 \left(\mu_1^*\right)^2}}}{x} \tag{34}$$

where f_∞ is some parameter. The next step is substituting the first approximation $f_1^*(x)$ into equation (13) for finding the regular exceptional solution $\phi_1^*(x)$ by m_1^* then $\phi_1^*(x)$ will be substituted into equation (14) for finding the regular exceptional solution $f_2^*(x)$ by $\mu = \mu_2^*$ and so on.

The result of these calculations is presented on Fig's.3, 4 and Table 1. We see that there is the convergence $\phi_i^*(x) \to \phi^*(x)$, $f_i^*(x) \to f^*(x)$, $m_i^* \to m^*$ and $\mu_i^* \to \mu^*$ where $f^*(x), \phi^*(x)$ are the eigenstates and m^*, μ^* are eigenvalues of nonlinear eigenvalue problem (11) (12).

The verification of the presented numerical method was done for the soliton solution, for details see Appendix B.

i	1	2	3	4
m_i^*	1.8374351...	1.594328...	1.6186108...	1.61823766...
μ_i^*	1.492105312...	1.4938287...	1.4921473...	1.4921473...

Table 1: The iterative parameters m_i^* and μ_i^*.

5 The properties of solution

In this section we would like to describe the properties of the derived solution. It is easy to see that the asymptotical behaviour of the regular solution is

$$\phi^*(x) \approx m^* + \beta \frac{e^{-x\sqrt{2\lambda_1 (m^*)^2}}}{x} \tag{35}$$

$$f^*(x) \approx f_\infty \frac{e^{-x\sqrt{(m^*)^2 - \lambda_2 (\mu^*)^2}}}{x} \tag{36}$$

where m^* and μ^* are the parameters derived in the coarse of iterative solution of equations (13) (14). The energy density of the presented solution is

$$\varepsilon(r) = \frac{1}{g^2}\left[\phi'^2(r) + f'^2(r) + \frac{\lambda_1}{2}\left(\phi^2(r) - m^{*2}\right) + \frac{\lambda_2}{2}f^2(r)\left(f^2(r) - 2\mu^{*2}\right) + f^2(r)\phi^2(r)\right] \tag{37}$$

here $\lambda_{1,2}, m^*$ and μ^* are redefined according the remark after eq. (12) and we add the constant term $-\lambda_2\mu^2/4$ for the finiteness of the full energy. Thus the glueball energy is

$$W = \frac{4\pi}{g^2}\phi_0\int\limits_0^\infty x^2\left[f'^2 + \phi'^2 + \frac{\lambda_1}{2}\left(\phi^2 - m^{*2}\right)^2 + \frac{\lambda_2}{2}f^2\left(f^2 - 2\mu^{*2}\right) + f^2\phi^2\right]dx = \frac{4\pi}{g^2}\phi_0 I_1\left(\lambda_{1,2}, m^*, \mu^*\right) \tag{38}$$

here we have redefined r, f and ϕ according to remark before eq. (13). The quantity ϕ_0^{-1} defines the radius of flux tube since the dimensionless variables x for flux tube is $x = \rho\phi_0$. The profile of the energy density is presented on Fig. 5. The numerical calculations for the dimensionless integral I_1 gives

$$I_1 = \int\limits_0^\infty x^2\left[f'^2 + \phi'^2 + \frac{\lambda_1}{2}\left(\phi^2 - m^{*2}\right)^2 + \frac{\lambda_2}{2}f^2\left(f^2 - 2\mu^{*2}\right) + f^2\phi^2\right]dx \approx 6.28. \tag{39}$$

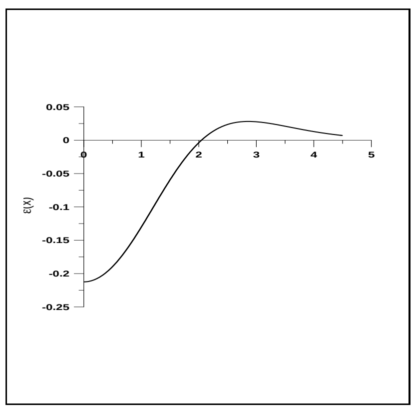

Figure 5: The profile of the energy density.

The next what we can do is the calculation the ratio of the energy glueball and string tension. On the basis of similar ideas presented here a flux tube solution was found in Ref. [8]. The basic idea is that the quantized SU(3) gauge potential $A^B_\mu, B = 1, 2, \ldots, 8$ can be splitted on two pieces: (a) the potential components $A^a_\mu, a = 1, 2, 3$ belong to the SU(2) subgroup, (b) the potential components $A^m_\mu, m = 4, 5, 6, 7, 8$ are coset componets. For the flux tube case it is supposed that A^a_μ are almost classical degrees of freedom but A^m_μ componets (analogously to glueball case) are completely quantum degrees of fredom. 2 and 4-points Green's functions for the A^m_μ components are expressed via a scalar field ϕ^a (what is similar to presented here glueball case). The assumptions which are similar to glueball case lead to a numerical solution describing a flux tube filled with the longitudinal color electric field. The linear energy density (or string tension) is

$$\begin{aligned}\sigma =& \frac{\pi}{g^2}\int\limits_0^\infty \rho\left[f'^2 + v'^2 + \phi'^2 + v^2 f^2 + v^2\phi^2 + f^2\phi^2 + m_1^* f^2 - m_2^* v^2 + \frac{\lambda}{2}\left(\phi^2 - \phi^*_\infty\right)^2\right]d\rho = \\ & \frac{\pi}{g^2}\phi_0^2 I_2\left(\lambda, \phi_\infty, m^*_{1,2}\right)\end{aligned} \tag{40}$$

where

$$A^1_t(\rho) = \frac{f(\rho)}{g}; \quad A^2_z(\rho) = \frac{v(\rho)}{g}; \quad \phi^3(\rho) = \frac{\phi(\rho)}{g}. \tag{41}$$

The numerical calculations give $I_2 \approx 0.63$.

The flux of the electric field is

$$\Phi = \int E_z^3 ds = 2\pi \int_0^\infty \rho \frac{f(\rho)v(\rho)}{g} d\rho = \frac{2\pi}{g} \int_0^\infty x f(x) v(x) dx = \frac{2\pi}{g} I_3 \left(\lambda, \phi_\infty, m^*_{1,2}\right) \tag{42}$$

where $E_z^3 = fv/g$ is the longitudinal color electric field. Eq. (42) shows that (like to Coulomb law) the flux of the electric field is proportional to a color charge defined as $q = 1/g$. But of course there is a dimensionless correction I_3 coming from the nonlinearity of the theory. The numerical calculations give $I_3 \approx 0.79$.

Let us consider the ratio

$$\frac{W}{\sqrt{\sigma}} = \sqrt{\frac{4\pi}{g^2}} \frac{2I_1}{\sqrt{I_2}} \approx 5 \tag{43}$$

here we take into account the value of dimensionless constant $g^2/4\pi \approx 10$. It is necessary to note that for the ratio (43) we consider the case when the scalar field ϕ has the same magnitudes at the center of flux tube and glueball. It can be compared with the lattice calculations [13] where this quantity is presented as $W/\sqrt{\sigma} \approx 3.64$ for 0^{++} glueball.

It is interesting to consider the dimensionless ratio

$$\frac{\sqrt{\sigma}\Phi}{W} = \frac{\sqrt{\pi}}{4} \frac{\sqrt{I_2} I_3}{I_1} \approx 0.04 \tag{44}$$

which tell us that the ratio (43) is proportional to the flux of electric field.

Now we would like to calculate the angular momentum of the glueball in the offered model. The angular momentum operator is

$$\widehat{\vec{M}} = \int \left[\vec{r} \times \left[\hat{\vec{E}}^A \times \hat{\vec{H}}^A\right]\right] dV \tag{45}$$

where $\hat{E}_i^B = \hat{F}_{0i}^B = \partial_0 \hat{A}_i^B - \partial_i \hat{A}_0^B + g f^{BCD} \hat{A}_0^C \hat{A}_i^D$ is the the operator of the color electric field; $\hat{H}_i^B = \epsilon_{ijk} \hat{F}^{Bjk}$ is the operator of the color magnetic field, and $\hat{F}_{jk}^B = \partial_j \hat{A}_k^B - \partial_k \hat{A}_j^B + g f^{BCD} \hat{A}_j^C \hat{A}_k^D$; $i, j, k = 1, 2, 3$. Let us consider

$$\begin{aligned}
\hat{m}_i = \left[\vec{r} \times \left[\hat{\vec{E}}^A \times \hat{\vec{H}}^A\right]\right]_i &= \epsilon_{ijk} \epsilon_{klm} \epsilon_{mpq} x^j \hat{F}_{0l}^A \hat{F}^{Apq} = \\
\epsilon_{ijk} \epsilon_{klm} \epsilon_{mpq} x^j &\left(\partial_0 \hat{A}_i^B - \partial_i \hat{A}_0^B + g f^{BCD} \hat{A}_0^C \hat{A}_i^D\right) \left(\partial_j \hat{A}_k^B - \partial_k \hat{A}_j^B + g f^{BCD} \hat{A}_j^C \hat{A}_k^D\right)
\end{aligned} \tag{46}$$

One can show that the expectation value

$$\langle \hat{m}_i \rangle = 0 \tag{47}$$

as in the considered case either $\partial_0(\cdots) \equiv 0$ or $\left\langle \hat{A}_0^B \hat{A}_i^C \cdots \right\rangle \equiv 0$ in the consequence of the presence the factor $\eta_{\mu\nu}$ in the assumptions (4) and (5). It means that the spin of the presented glueball model is zero. It is interesting to note that there is an opinion [14] that pure glueball can only be spin 0.

From this consideration immediately we see that in this approach the glueball with nonzero spin probably can be derived using ansätz similar (4) but with nonzero correlation between A_0^B and A_i^C.

Another interesting possibility for the future investigations is the derivation of a mass spectrum. For this we see two ways: the first one is the search of excited states on the basis of ansätz (4) for the 2-point Green's function; the second one is the search an another ansätz for the 2-point Green's function which gives the glueball with another mass. The preliminary investigations show that probably excited states can not be derived using the presented iterative method. Thus the derivation of glueball mass spectrum in the nonperturbative approach is the complicated problem and it is the goal of the future investigations.

Finally, we would like to touch upon the connection between our solution and Derrick's Theorem [15]. This theorem tells us that in 3 spatial dimensions the scalar field theory with nonnegative potential do not have absolute stable solutions with finite energy which means that at the infinity the solution must tend to a global minimum where $V(\textit{global minimum}) = 0$. But the potential for the interacting scalar fields of (6) has global and local minima. Our solution is in one of the local minima. The Derrick's Theorem tells us that if we add two constant terms $\frac{\lambda_1}{4}(\phi_0^a \phi_0^a)^2$ and $\frac{\lambda_2}{4}(\phi_0^m \phi_0^m)^2$ to the potential in eq. (6) we will have the following potential

$$\begin{aligned}
&V = \frac{\lambda_1}{4} \left[\phi^a \phi^a - \phi_0^a \phi_0^a\right]^2 + \frac{\lambda_2}{4} \left[\phi^m \phi^m - \phi_0^m \phi_0^m\right]^2 + \left(\phi^a \phi^a\right)\left(\phi^m \phi^m\right), \\
&V\,(\text{global minimum}) = 0, \\
&V\,(\text{local minimum}) > 0.
\end{aligned} \tag{48}$$

In this case the presented solution (in the full agreement with the Derrick's Theorem) will have the infinite energy. If we add only one constant term $\frac{\lambda_2}{4}(\phi_0^m \phi_0^m)^2$ we will have the following potential

$$
\begin{aligned}
&V = \frac{\lambda_1}{4}\left[\phi^a \phi^a - \phi_0^a \phi_0^a\right]^2 + \frac{\lambda_2}{4}\phi^m \phi^m \left[\phi^m \phi^m - 2\phi_0^m \phi_0^m\right] + \left(\phi^a \phi^a\right)\left(\phi^m \phi^m\right), \\
&V\,(\text{global minimum}) < 0, \\
&V\,(\text{local minimum}) = 0.
\end{aligned}
\tag{49}
$$

In this case the energy of the glueball solution is finite but the stability of the solution have to be investigated in the future works.

After the discussion of the properties of derived solution one can see that the presented model is the model of glueball with zero spin where some combination of fields ϕ^a and ϕ_m push out each other.

6 Physical discussion and conclusions

In this letter we discuss the glueball solution presented on Fig's.3, 4. This scalar model of glueball is derived with the assumptions that: (a) 2 and 4-points Green's functions of the SU(3) gauge potential can be approximately expressed via a scalar field; (b) the scalar fields components with a small subgroup indices belonging to $SU(2) \in SU(3)$ can have the different qualitative behaviour in some physical situations (in flux tube and glueball) in comparison with the scalar components which indices belong to the coset $SU(3)/SU(2)$. As the consequence we see that in this case exists a blob of the quantized SU(3) gauge field and the coset components push out the SU(2) components of the scalar field that is like to the Meissner effect in superconductivity. Such solution can be interpreted as the glueball in a medium. It follows from the fact that 2 and 4-points Green's functions of ϕ^m are nonzero at the infinity.

Remarkably that similar situation exists in a flux tube solution obtained in Ref.[8]. There is only one essential difference between flux tube and glueball solutions: in the flux tube solution the gauge potential components belonging to the small subgroup SU(2) in the first approximation can be considered as classical degrees of freedom that allows to exist a longitudinal color electric field directed from quark to antiquark.

Let us to underscore that in this interpretation the derived bubble of the quantized SU(2) components live in the sea of the quantized coset components. But we can present an another interpretation of this solution in which both components exist in vacuum.

One can unite two assumptions about 2-point Green's function (4) and the term breaking the gauge invariance $\phi_0^A \phi_0^A$ in one

$$
\left\langle \widehat{A}^B_\alpha(x) \widehat{A}^C_\beta(y) \right\rangle \approx -\eta_{\alpha\beta}\left[f^{BAd} f^{CAe} \phi^d(x) \phi^e(y) + f^{BAm} f^{CAn}\left(\phi^m(x)\phi^n(y) - \phi_0^m \phi_0^n\right)\right]. \tag{50}
$$

Let us remind that the indices $d, e = 1, 2, 3$ and $m, n = 4, 5, 6, 7, 8$. In this case

$$
\left\langle \widehat{A}^a_\alpha(x) \widehat{A}^a_\beta(x) \right\rangle \approx -\eta_{\alpha\beta}\left[\sum_c \left(f^{acd}\right)^2 \left(\phi^d(x)\right)^2 + \sum_m \left(f^{amn}\right)^2 \left(\phi^n(x) - \phi_0^n\right)^2\right], \tag{51}
$$

$$
\begin{aligned}
\left\langle \widehat{A}^m_\alpha(x) \widehat{A}^m_\beta(x) \right\rangle \approx -\eta_{\alpha\beta}\Bigg[&\sum_n \left(f^{mna}\right)^2 \left(\phi^a(x)\right)^2 + \sum_a \left(f^{man}\right)^2 \left(\phi^n(x) - \phi_0^n\right)^2 + \\
&\sum_n \left(f^{mn8}\right)^2 \left(\phi^8(x) - \phi_0^8\right)^2\Bigg]
\end{aligned}
\tag{52}
$$

which describe the variance of nonlinear oscillations of the gauge field and they are nonzero inside of the bubble only. It means that the quantized field is concentrated in this region. By such a manner the correlation between $A^B(x)$ and $A^C(x), B \neq C$ components are nonzero in the same region. Consequently one can say that in this approach the quantized field SU(3) fields is concentrated in the bubble and can be interpreted as glueball in the vacuum.

The presented here approach to the QCD is similar to a field correlator method [11] with one difference: in our approach there is dynamical equations for the Green's functions which are derived from the SU(3) Lagrangian.

This approach to the Green's functions which can be approximately considered as scalar fields (or a condensate) may have interesting applications for gravity where scalar fields have various applications: inflation, boson stars, non-Abelian black holes and so on. Our approach allows us to speculate that the nonperturbative quantum effects can be very important in some gravitational phenomenon.

7 Acknowledgments

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A The effective Lagrangian

In order to derive equations describing the quantized field we average the Lagrangian over a quantum state $|Q\rangle$

$$\begin{aligned}\left\langle Q\left|\widehat{\mathcal{L}}\right|Q\right\rangle = \left\langle \widehat{\mathcal{L}}\right\rangle = \frac{1}{2}\left\langle\left(\partial_\mu \widehat{A}^B_\nu\right)\left(\partial^\mu \widehat{A}^{B\nu}\right) - \left(\partial_\mu \widehat{A}^B_\nu\right)\left(\partial^\nu \widehat{A}^{B\mu}\right)\right\rangle + \\ \frac{1}{2} g f^{BCD}\left\langle\left(\partial_\mu \widehat{A}^B_\nu - \partial_\nu \widehat{A}^B_\mu\right)\widehat{A}^{C\mu}\widehat{A}^{D\nu}\right\rangle + \frac{1}{4} g^2 f^{BC_1D_1} f^{BC_2D_2}\left\langle \widehat{A}^{C_1}_\mu \widehat{A}^{D_1}_\nu \widehat{A}^{C_2\mu}\widehat{A}^{D_2}\nu\right\rangle\end{aligned} \tag{53}$$

Schematically we have the following 2, 3 and 4-points Green's functions: $\left\langle(\partial A)^2\right\rangle$, $\left\langle(\partial A)\, A^2\right\rangle$ and $\left\langle(A)^4\right\rangle$. At first we introduce the 2-point Green's function

$$\left\langle \widehat{A}^B_\alpha(x)\widehat{A}^C_\beta(y)\right\rangle = \mathcal{G}^{BC}_{\alpha\beta}(x,y) \tag{54}$$

The first term on the rhs of equation (53) is

$$\left(\partial_\mu \widehat{A}^B_\nu(x)\right)\left(\partial^\mu \widehat{A}^{B\nu}(x)\right) = \partial_{x\mu}\partial^\mu_y \left(\widehat{A}^B_\nu(x)\right)\left(\widehat{A}^{B\nu}(y)\right)\Big|_{y\to x} = \eta^{\alpha\beta}\partial_{x\mu}\partial^\mu_y \mathcal{G}^{BB}_{\alpha\beta}(x,y)\Big|_{y\to x}. \tag{55}$$

For the simplicity we consider the case with $x_0 = y_0$. For this Green's function we use so called one-function approximation [9]

$$\mathcal{G}^{AB}_{\alpha\beta}(x,y) \approx -\eta_{\alpha\beta} f^{ACD} f^{BCE}\phi^D(x)\phi^E(y) \tag{56}$$

where $\phi^A(x)$ is the scalar field which describes the 2-point Green's function. Physically this approximation means that quantum properties of the field A^B_μ can be approximately described by a scalar field $\phi^B(x)$, i.e. in this approximation the Lorentz index μ is not very important. Taking into account this approximation we have

$$\left\langle\left(\partial_\mu \widehat{A}^B_\nu\right)\left(\partial^\mu \widehat{A}^{B\nu}\right)\right\rangle = -\eta^\nu_\nu f^{BAC} f^{BAD}\left(\partial_\mu\phi^C\right)\left(\partial^\mu\phi^D\right) = -12\left(\partial_\mu\phi^A\right)\left(\partial^\mu\phi^A\right) \tag{57}$$

and

$$\left\langle\left(\partial_\mu \widehat{A}^B_\nu\right)\left(\partial^\nu \widehat{A}^{B\mu}\right)\right\rangle = -3\left(\partial_\mu\phi^A\right)\left(\partial_\mu\phi^A\right), \tag{58}$$

Later we suppose that the odd Green's functions can be expressed as the sum of the following products

$$\left\langle \widehat{A}^B_\alpha(x)\widehat{A}^C_\beta(y)\widehat{A}^D_\gamma(z)\right\rangle \approx \left\langle \widehat{A}^B_\alpha(x)\widehat{A}^C_\beta(y)\right\rangle\left\langle \widehat{A}^D_\gamma(z)\right\rangle + (\text{other permutations}) = 0 \tag{59}$$

as $\langle \widehat{A}^B_\alpha(x)\rangle = 0$. It gives us

$$\left\langle\left(\partial_\mu \widehat{A}^B_\alpha(x)\right)\widehat{A}^C_\beta(x)\widehat{A}^D_\gamma(x)\right\rangle = \partial_{x\mu}\left\langle \widehat{A}^B_\alpha(x)\widehat{A}^C_\beta(y)\widehat{A}^D_\gamma(z)\right\rangle\Big|_{y,z\to x} = 0 \tag{60}$$

and consequently in our approximation

$$\left\langle\left(\partial_\mu \widehat{A}^B_\nu - \partial_\nu \widehat{A}^B_\mu\right)\widehat{A}^{C\mu}\widehat{A}^{D\nu}\right\rangle = 0. \tag{61}$$

For the last quartic term on the rhs of equation (53) we assume the following approximation

$$\begin{aligned}\left\langle \widehat{A}^B_\alpha(x)\widehat{A}^C_\beta(y)\widehat{A}^D_\gamma(z)\widehat{A}^R_\delta(u)\right\rangle \approx \left\langle \widehat{A}^B_\alpha(x)\widehat{A}^C_\beta(y)\right\rangle\left\langle \widehat{A}^D_\gamma(z)\widehat{A}^R_\delta(u)\right\rangle + \\ \left\langle \widehat{A}^B_\alpha(x)\widehat{A}^D_\gamma(z)\right\rangle\left\langle \widehat{A}^C_\beta(y)\widehat{A}^R_\delta(u)\right\rangle + \left\langle \widehat{A}^B_\alpha(x)\widehat{A}^R_\gamma(u)\right\rangle\left\langle \widehat{A}^C_\beta(y)\widehat{A}^D_\gamma(z)\right\rangle.\end{aligned} \tag{62}$$

In fact it is the assumption that 4-point Greens function is the product of two 2-points Green's function. In this approximation the lhs of (62) is

$$\begin{aligned}&\left\langle \widehat{A}^B_\mu(x)\widehat{A}^C_\nu(x)\widehat{A}^{D\mu}(x)\widehat{A}^{R\nu}(u)\right\rangle = \\ &\lambda_{1,2;(P_{1,2},Q_{1,2})}\left(f^{BE_1P_1} f^{CE_1Q_1}\phi^{P_1}(x)\phi^{Q_1}(x)\right)\left(f^{DE_2P_2} f^{RE_2Q_2}\phi^{P_2}(x)\phi^{Q_2}(x)\right)\eta_{\mu\nu}\eta^{\mu\nu} + \\ &\lambda_{1,2;(P_{1,2},Q_{1,2})}\left(f^{BE_1P_1} f^{DE_1Q_1}\phi^{P_1}(x)\phi^{Q_1}(x)\right)\left(f^{CE_2P_2} f^{RE_2Q_2}\phi^{P_2}(x)\phi^{Q_2}(x)\right)\eta^\mu_\mu\eta^\nu_\nu + \\ &\lambda_{1,2;(P_{1,2},Q_{1,2})}\left(f^{BE_1P_1} f^{RE_1Q_1}\phi^{P_1}(x)\phi^{Q_1}(x)\right)\left(f^{CE_2P_2} f^{DE_2Q_2}\phi^{P_2}(x)\phi^{Q_2}(x)\right)\eta^\nu_\mu\eta^\mu_\nu\end{aligned} \tag{63}$$

where $\lambda_{1,2;(P_{1,2},Q_{1,2})}$ is some parameter depending on the values of the indices $P_{1,2}, Q_{1,2}$

$$\lambda_{1,2;(P_{1,2},Q_{1,2})} = \begin{cases} \lambda_1, & \text{if all indices } P_{1,2}, Q_{1,2} = 1,2,3, \\ \lambda_2, & \text{if all indices } P_{1,2}, Q_{1,2} = 4,5,6,7,8, \\ 1, & \text{otherwise} \end{cases} \tag{64}$$

where $\lambda_{1,2}$ are some parameters. Introducing this index $\lambda_{1,2;(P_{1,2},Q_{1,2})}$ we would like to say that the presented approximate quantization procedure is a little different for the scalar field components belonging to the small subgroup $SU(2) \in SU(3)$ and the coset $SU(3)/SU(2)$. In this case

$$\begin{aligned} \left(f^{BE_1P_1} f^{CE_1Q_1} \phi^{P_1} \phi^{Q_1}\right) \left(f^{DE_2P_2} f^{RE_2Q_2} \phi^{P_2} \phi^{Q_2}\right) = \\ \lambda_1 \left(f^{BE_1a} f^{CE_1b} \phi^a \phi^b\right) \left(f^{DE_2c} f^{RE_2d} \phi^c \phi^d\right) + \lambda_2 \left(f^{BE_1m} f^{CE_1n} \phi^m \phi^n\right) \left(f^{DE_2p} f^{RE_2q} \phi^p \phi^q\right) + \\ \text{(other terms)} \end{aligned} \tag{65}$$

The calculations show that

$$f^{ABC} f^{ADR} \left(f^{BE_1P_1} f^{CE_1Q_1} \phi^{P_1} \phi^{Q_1}\right) \left(f^{DE_2P_2} f^{RE_2Q_2} \phi^{P_2} \phi^{Q_2}\right) = 0, \tag{66}$$

$$f^{ABC} f^{ADR} \left(f^{BE_1a} f^{CE_1b} \phi^a \phi^b\right) \left(f^{DE_2c} f^{RE_2d} \phi^c \phi^d\right) = 0, \tag{67}$$

$$f^{ABC} f^{ADR} \left(f^{BE_1m} f^{CE_1n} \phi^m \phi^n\right) \left(f^{DE_2p} f^{RE_2q} \phi^p \phi^q\right) = 0. \tag{68}$$

Consequently

$$\lambda_{1,2;(P_{1,2},Q_{1,2})} \left(f^{BE_1P_1} f^{CE_1Q_1} \phi^{P_1}(x) \phi^{Q_1}(x)\right) \left(f^{DE_2P_2} f^{RE_2Q_2} \phi^{P_2}(x) \phi^{Q_2}(x)\right) \eta_{\mu\nu} \eta^{\mu\nu} = 0. \tag{69}$$

The similar calculations show that

$$\begin{aligned} f^{ABC} f^{ADR} \left(f^{BE_1P_1} f^{DE_1Q_1} \phi^{P_1}(x) \phi^{Q_1}(x)\right) \left(f^{CE_2P_2} f^{RE_2Q_2} \phi^{P_2}(x) \phi^{Q_2}(x)\right) &= \\ \frac{27}{8} \left(\phi^a \phi^a + \phi^m \phi^m\right)^2, \end{aligned} \tag{70}$$

$$f^{ABC} f^{ADR} \left(f^{BE_1a} f^{DE_1b} \phi^a \phi^b\right) \left(f^{CE_2c} f^{RE_2d} \phi^c \phi^d\right) = \frac{27}{8} \left(\phi^a \phi^a\right)^2, \tag{71}$$

$$f^{ABC} f^{ADR} \left(f^{BE_1m} f^{DE_1n} \phi^m \phi^n\right) \left(f^{CE_2p} f^{RE_2q} \phi^p \phi^q\right) = \frac{27}{8} \left(\phi^m \phi^m\right)^2. \tag{72}$$

Consequently

$$\begin{aligned} \lambda_{1,2;(P_{1,2},Q_{1,2})} \left(f^{BE_1P_1} f^{DE_1Q_1} \phi^{P_1}(x) \phi^{Q_1}(x)\right) \left(f^{CE_2P_2} f^{RE_2Q_2} \phi^{P_2}(x) \phi^{Q_2}(x)\right) = \\ \frac{27}{8} \lambda_1 \left(\phi^a \phi^a\right)^2 + \frac{27}{8} \lambda_2 \left(\phi^m \phi^m\right)^2 + \frac{27}{4} \left(\phi^a \phi^a\right) \left(\phi^m \phi^m\right). \end{aligned} \tag{73}$$

Analogously

$$\begin{aligned} \lambda_{1,2;(P_{1,2},Q_{1,2})} \left(f^{BE_1P_1} f^{RE_1Q_1} \phi^{P_1}(x) \phi^{Q_1}(x)\right) \left(f^{CE_2P_2} f^{DE_2Q_2} \phi^{P_2}(x) \phi^{Q_2}(x)\right) = \\ \frac{27}{8} \lambda_1 \left(\phi^a \phi^a\right)^2 + \frac{27}{8} \lambda_2 \left(\phi^m \phi^m\right)^2 + \frac{27}{4} \left(\phi^a \phi^a\right) \left(\phi^m \phi^m\right). \end{aligned} \tag{74}$$

Finally for $x = y = z = u$ the quartic term is

$$f^{ARB} f^{ACD} \left\langle \widehat{A}^R_\mu \widehat{A}^B_\nu \widehat{A}^{C\mu} \widehat{A}^D{}_\nu \right\rangle = \frac{81}{2} \lambda_1 \left(\phi^a \phi^a\right)^2 + \frac{81}{2} \lambda_2 \left(\phi^m \phi^m\right)^2 + 81 \left(\phi^a \phi^a\right) \left(\phi^m \phi^m\right). \tag{75}$$

Therefore we have the following effective Lagrangian describing 2 and 4-points Green's functions

$$\mathcal{L}_{eff} = -\frac{9}{2} \left(\partial_\mu \phi^A\right) \left(\partial^\mu \phi^A\right) + \frac{g^2}{4} \left[\frac{81}{2} \lambda_1 \left(\phi^a \phi^a\right)^2 + \frac{81}{2} \lambda_2 \left(\phi^m \phi^m\right)^2 + 81 \left(\phi^a \phi^a\right) \left(\phi^m \phi^m\right)\right]. \tag{76}$$

If we redefine $\phi^a \to 2\phi^a/(3g)$ and $\lambda_{1,2} \to \lambda_{1,2}/2$ we will have the ordinary Lagrangian for the scalar field

$$\frac{g^2}{4} \mathcal{L}_{eff} = -\frac{1}{2} \left(\partial_\mu \phi^A\right) \left(\partial^\mu \phi^A\right) + \frac{\lambda_1}{4} \left(\phi^a \phi^a\right)^2 + \frac{\lambda_2}{4} \left(\phi^m \phi^m\right)^2 + \left(\phi^a \phi^a\right) \left(\phi^m \phi^m\right). \tag{77}$$

Now it is necessary to do an essential remark. The SU(3) Lagrangian for the gauge group A^B_μ is very nonlinear: it has A^4 terms. It is well known [10] that in $\lambda\phi^4$ theory the similar nonlinearity give rise

to an additional term to potential term. One can suppose that the similar situation takes place in this situation, too. Here we suppose that the nonlinear terms like A^4 leads to the appearance of some term in the initial Lagrangian. For the simplicity we assume that the mass term will appear. Thus the final form of the effective Lagrangian is

$$\begin{aligned}\frac{g^2}{4}\mathcal{L}_{eff} = -\frac{1}{2}\left(\partial_\mu \phi^A\right)^2 + \frac{\lambda_1}{4}\left[\phi^a\phi^a - \phi^a_0\phi^a_0\right]^2 - \frac{\lambda_1}{4}\left(\phi^a_0\phi^a_0\right)^2 + \\ \frac{\lambda_2}{4}\left[\phi^m\phi^m - \phi^m_0\phi^m_0\right]^2 - \frac{\lambda_2}{4}\left(\phi^m_0\phi^m_0\right)^2 + \left(\phi^a\phi^a\right)\left(\phi^m\phi^m\right)\end{aligned} \tag{78}$$

where ϕ^A_0 are some constants. In this situation the field equations for the approximate scalar description of the QCD are

$$\partial_\mu\partial^\mu\phi^a = \phi^a\left[2\phi^m\phi^m + \lambda_1\left(\phi^a\phi^a - \phi^a_0\phi^a_0\right)\right], \tag{79}$$

$$\partial_\mu\partial^\mu\phi^m = \phi^m\left[2\phi^a\phi^a + \lambda_2\left(\phi^m\phi^m - \phi^m_0\phi^m_0\right)\right]. \tag{80}$$

In conclusion we have to note that this procedure for the approximate calculations of 2 and 4-points Green's functions should be some approximation for an exact procedure which obtains *all* Green's functions bu a nonperturbative manner. At first such procedure was offered by Heisenberg for the quantization of a nonlinear spinor field [12] and later was applied for the QCD [9].

B The numerical calculations of the soliton

For the validation of the presented method of solving the nonlinear equations (11) (12) we choose a soliton solution. The corresponding equation is

$$\frac{d^2y}{dx^2} = y'' = y\left(1 - y^2\right). \tag{81}$$

The solution is

$$y(x) = \frac{\sqrt{2}}{\cosh x}. \tag{82}$$

We rewrite the (81) equation in the form of the Schrödinger equation

$$-y'' + yV_{eff} = -\lambda y \tag{83}$$

where $V_{eff} = -y^2$ and $\lambda = 1$. It shows us that the regular solution exists only for a discrete spectrum of "energy level" λ. We will solve this equation by an iterative procedure. At first we have the equation

$$-y_1'' + y_1\left(-y_0^2\right) = -\lambda_1 y_1 \tag{84}$$

for the first approximation $y_1(x)$ and where λ_1 is the first approximation for the λ. For the numerical solution we choose the null approximation as

$$y_0 = \frac{\sqrt{2}}{\cosh\left(\frac{x}{2}\right)}. \tag{85}$$

The typical solution for the arbitrary values of the parameter λ_1 is presented on Fig.6. This picture shows us that there is a value λ_1^* for which the solution is exceptional one. One can find this exceptional solution choosing the appropriate value of the "energy level" λ_1^*. After which an exceptional solution $y_1^*(x)$ is substituted into equation for the second approximation $y_2(x)$

$$-y_2'' - y_2\left(y_1^*\right)^2 = -\lambda_2 y_2 \tag{86}$$

and so on. The result is presented on Table 2 and Fig.7. One can see that $\lambda_i^* \to 1$ and $y_i^*(x)$ is convergent to $y^*(x)$.

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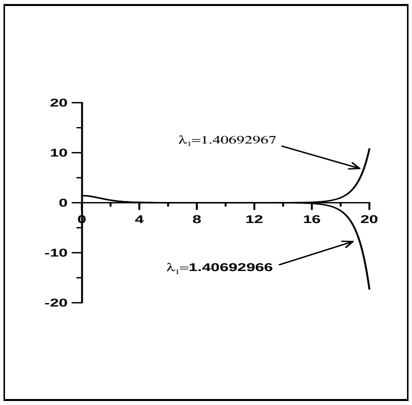

Figure 6: The singular solutions for the soliton equation.

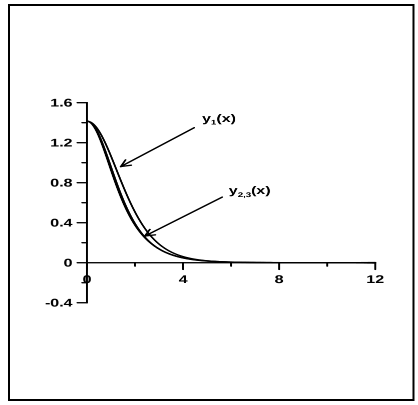

Figure 7: The iterative functions $y_{1,2,3}$.

i	1	2	3
λ_i^*	1.406929666...	1.148564915...	1.03968278...

Table 2: The iterative values of the parameter λ.

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